Geometric Analysis Conference

Abstracts

2014/07/08, 15:00 — 16:00
Cheeger, Courant Institute

Regularity of Einstein Manifolds and the Codimension 4 Conjecture

This talk concerns joint work with Aaron Naber. Let X denote the Gromov-Hausdorff limit of a noncollapsing sequence of Riemannian manifolds with bounded Ricci curvature. Our main result is that X is smooth away from a closed subset of Hausdorff codimension 4. We combine this with ideas of quantitative stratification to prove a priori L^q curvature estimates for all $q < 2$ and corresponding estimates on the set of points with small harmonic radius. In the Einstein case, we improve this to estimates on the regularity scale. As an application, we prove a conjecture of Anderson stating that the collection of 4-manifolds bound Ricci curvature, volume bounded from below and diameter bounded from above contains at most a definite number of diffeomorphism classes. A local version of this is used to show that noncollapsed 4-manifolds with bounded Ricci curvature have a priori L^2 Riemannian curvature estimates.

2014/07/08, 09:00 — 10:00
Codá Marques, Instituto de Matemática Pura e Aplicada

Existence of infinitely many minimal hypersurfaces in positive Ricci curvature

In the early 1980s, S. T. Yau conjectured that any compact Riemannian three-manifold admits an infinite number of closed immersed minimal surfaces. In this talk I will describe joint work with André Neves, in which we use min-max theory for the area functional to prove this conjecture in the positive Ricci curvature setting. More precisely, we show that every compact Riemannian manifold with positive Ricci curvature and dimension at most seven contains infinitely many smooth, closed, embedded minimal hypersurfaces.

2014/07/08, 11:45 — 12:45
Girão, Instituto Superior Técnico

On the global uniqueness for the Einstein-Maxwell-scalar field system with a cosmological constant.

This talk is dedicated to the following problem: given spherically symmetric characteristic initial data for the Einstein-Maxwell-scalar field system with a cosmological constant Λ, with the data on the outgoing initial null hypersurface given by a subextremal Reissner-Nordström black hole event horizon, study the future extendibility of the corresponding maximal globally hyperbolic development (MGHD) as a "suitably regular" Lorentzian manifold.

First, we establish the well posedness of the characteristic problem. Second, we study the stability of the radius function at the Cauchy horizon. Third, we show that, depending on the decay rate of the initial data, mass inflation may or may not occur. When the mass is controlled, it is possible to obtain continuous extensions of the metric across the Cauchy horizon with square integrable Christoffel symbols. Under slightly stronger conditions, we can bound the gradient of the scalar field. This allows the construction of (non-isometric) extensions which are classical solutions of the Einstein equations.

These results comprise the trilogy arXiv:1406.7245, arXiv:1406.7253 and arXiv:1406.7261 by J. Costa, P. Girão, J. Natário and J. Silva.

2014/07/09, 09:00 — 10:00
Hitchin, Oxford University

Folded hyperkähler manifolds

This talk is motivated by trying to extend the theory of Higgs bundles on a Riemann surface from the group $SU(2)$ to the group of symplectic diffeomorphisms of the 2-sphere. A type of degenerate hyperkähler metric on a 2-sphere bundle over the surface emerges which is a particular case of the folded symplectic and Kähler structures which already appear in the literature. One such example, which is the model for various conjectures, is derived from the hyperbolic metric on the surface.

2014/07/10, 11:45 — 12:45
Ilmanen, ETH Zürich

Flow of Trivial Networks by Curvature in $\mathbb{R}^2$

We study the flow of networks with triple junctions from the viewpoint of weak solutions.

1. A "60-flow" is a weak (varifold) evolution of 1-dimensional figures in the plane by curvature, with junction angles that are constrained to be multiples of 60 degrees. We prove a compactness theorem.
2. We exhibit various singularities and tangent flows, including multi-bubble shrinkers, k-trees, cross singularities, spikes, and higher multiplicity examples.
3. Short-time existence of regular solutions of the **singular** initial value problem.
4. Multilayer regularity lemma.
5. Regularity of the "sudden vanishing" set.
6. Ultra-compactness for shrinkers.
7. Disturbing the bee's nest.

2014/07/08, 16:30 — 17:30
Kleiner, Courant Institute

Ricci flow through singularities

It has been a long-standing problem in geometric analysis to find a good definition of generalized solutions to the Ricci flow equation that would formalize the heuristic idea of flowing through singularities. I will discuss a notion in the 3-d case that has good analytical properties, enabling one to prove existence and compactness of solutions, as well as a number of structural results. It may also be used to partly address a question of Perelman concerning the convergence of Ricci flow with surgery to a canonical flow through singularities. This is joint work with John Lott.

2014/07/09, 10:30 — 11:30
LeBrun, Stony Brook University

Weyl Curvature, Einstein Metrics, and the Geometry of 4-Manifolds

The Weyl functional is a natural measure of the deviation of a compact Riemannian manifold from conformal flatness; in dimension n, it is by definition the $(n/2)$-norm of the Weyl curvature, raised to the power $n/2$. Its behavior turns out to display special features in dimension four that do not generalize to other dimensions. In particular, Einstein metrics are critical points of this functional in dimension 4, while the rather different class of "anti-self-dual" metrics are in fact absolute minimizers. In this talk, I will describe some recent results concerning the problem of determining whether certain Einstein metrics are absolute minimizers, with ramifications for the existence of new obstructions to the existence of anti-self-dual metrics.

2014/07/10, 15:00 — 16:00
Lee, City College of New York

The Penrose inequality for asymptotically locally hyperbolic spaces with nonpositive mass

In the asymptotically locally hyperbolic setting it is possible to have metrics with scalar curvature at least −6 and negative mass when the genus of the conformal boundary at infinity is positive. Using inverse mean curvature flow, we prove a Penrose inequality for these negative mass metrics. The motivation comes from a previous result of P. Chrusciel and W. Simon, which states that the Penrose inequality we prove implies a static uniqueness theorem for negative mass Kottler metrics. This is joint work with André Neves.

2014/07/07, 16:30 — 17:30
Minicozzi, Massachusetts Institute of Technology

Uniqueness of blowups and Lojasiewicz inequalities

I will talk about two new infinite dimensional Lojasiewicz inequalities and their application to resolve the long-standing question of uniqueness of blowups for mean curvature flow at all generic singularities and for mean convex mean curvature flow at all singularities. This is joint work with Toby Colding.

2014/07/09, 11:45 — 12:45
Naber, Northwestern University

Characterizations of Bounded Ricci Curvature on Smooth and Nonsmooth Spaces

In this talk we discuss several new estimates on manifold with bounded Ricci curvature, and in particular Einstein manifolds. In fact, the estimates are not only implied by bounded Ricci curvature, but turn out to be equivalent to bounded Ricci curvature. We will see that bounded Ricci curvature controls analysis on the path space $P(M)$ of a manifold in much the same way that lower Ricci curvature controls analysis on M. There are three distinct such characterizations given. The first is a gradient estimate that acts as an infinite dimensional analogue of the Bakry Emery gradient estimate on path space. The second is a $C^{1/2}$ Holder estimate on the time regularity of the martingale decomposition of functions on path space. For the third we consider the Ornstein-Uhlenbeck operator, a form of infinite dimensional laplace operator, and show that bounded Ricci curvature is equivalent to an appropriate spectral gap. One can use these notions to make sense of bounded Ricci curvature on abstract metric-measure spaces.

2014/07/08, 10:30 — 11:30
Nabutovsky, University of Toronto

Curvature-free estimates for solutions of variational problems in Riemannian geometry

In my survey talk I will discuss when it is possible to give curvature-free upper bounds for lengths/areas/volumes of the "simplest" solutions of some classical variational problems on Riemannian manifolds. These stationary objects will include three simple perodic geodesics on Riemannian 2-spheres, periodic geodesics, minimal hypersurfaces, and different geodesics between a fixed pair of points in closed Riemannian manifolds. A special emphasize will be made on open problems.

2014/07/07, 10:30 — 11:30
Nadirashvili, University of Marseille

Hessian equations and minimal cones

We discuss singular solutions of Hessian elliptic equations and its connections with minimal cones.

2014/07/10, 16:30 — 17:30
Riviere, ETH Zürich

Some results on the calculus of variations of Riemann surfaces

We shall present various aspects of the minimization of functionals for immersion into the spheres of surfaces under constrained conformal class. We will focus in particular on the variations of the Willmore and the area functional in the class of weak conformal immersions of given Riemann surfaces. We will in particular give a characterization of riemann tori into S^3 minimizing locally the conformal volume.

2014/07/10, 10:30 — 11:30
Ros, University of Granada

Overdetermined elliptic problems in the plane

We present some results in relationship with the classical question of Berestycki, Caffarelli and Nirenberg on bounded positive solutions of the overdetermined problem

$$\begin{cases} \Delta u + f(u) = 0 \text{ in } \Omega, \\ u = 0 \text{ and } \frac{\partial u}{\partial n} = \alpha \text{ on } \partial\Omega, \end{cases}$$

where Ω is a smooth planar domain bounded by a proper Jordan arc. It's a joint work with P. Sicbaldi.

2014/07/11, 11:45 — 12:45
Rotman, University of Toronto

Geometry of Riemannian 2-spheres

I will discuss some geometric inequalities that are valid for all Riemannian 2-spheres. For example, consider the following basic question: Suppose a simple closed curve c on a Riemannian 2-sphere M of diameter D can be contracted to a point in M over closed curves of length at most L. Is there a homotopy over loops based at some point of c that are short compared to L and D? The answer to this question is positive. (Joint with G. Chambers).

I will also prove that for any posititve k and any two points of M there exist at least k geodesics connecting them of length at most $22kD$. (Joint with A. Nabutovsky).

2014/07/11, 09:00 — 10:00
Rupflin, Leipzig University

Teichmüller harmonic map flow

We will discuss Teichmüller harmonic map flow which is designed to evolve surfaces towards critical points of the Area. As we will discuss, the flow is simply defined as L^2-gradient flow of the Dirichlet energy, considered as functional of both a map and a metric on the domain, but exploits the symmetries of this functional. As such, it enjoys the strong regularity properties known from harmonic map flow for as long as there is no degeneration of the domain in Teichmüller space. Indeed, we shall show that for non-positively curved targets the flow admits global smooth solutions which, along a sequence $t_i \to \infty$, change or decompose arbitrary initial maps into branched minimal immersions. A key aspect of the proof is a quantitative analysis of holomorphic quadradic differentials on degenerating surfaces and in particular a uniform Poincaré estimate for quadratic differentials valid on any hyperbolic surface of given genus. The presented results are joint work with Peter Topping.

2014/07/07, 09:00 — 10:00
Schoen, Stanford University

On the asymptotic behavior of asymptotically flat spacetimes

Asymptotically flat spacetimes model isolated systems in general relativity. The Einstein equations allow a great variety of spatial asymptotic behavior with the simplest being that of the Schwarzschild or Kerr solutions. In this lecture we will outline many of the known results concerning asymptotic behavior and describe a recent one (joint with A. Carlotto) which constructs solutions which are flat outside cones with small cone angle. This turns out to be optimal in a precise sense.

2014/07/10, 09:00 — 10:00
White, Stanford University

Boundary behavior of Mean Curvature Flow

We consider mean curvature flow of surfaces with boundaries. We discuss boundary regularity theorems and singularities, with applications to minimal surface theory.

2014/07/11, 10:30 — 11:30
Wickramasekera, University of Cambridge

The blow-up method for minimal surfaces and mean curvature flows

The blow-up method was pioneered by De Giorgi to prove regularity of sufficiently flat parts of an area minimizing hypersurface and by Simon to prove regularity of the singular sets of multiplicity 1 minimal submanifolds. This powerful technique has since been used in various forms and levels of sophistication to produce an extremely rich array of regularity results for various classes of geometric objects including multiplicity 1 (possibly non-minimizing) minimal submanifolds (Allard), area minimizing submanifolds of arbitrary codimension (Almgren), stable minimal hypersurfaces of arbitrary multiplicity (Schoen-Simon, speaker) and multiplicity 1 mean curvature flows (Brakke, Kasai-Tonegawa). This method, in its strongest form, shows that whenever a possibly singular object in such a class is weakly close to a given homogeneous element in the class, it decays, upon rescaling, to a unique tangent object.

We will discuss two recent implementations of the blowup method. Both produce asymptotics near singularities, and the end results are:

1. uniqueness of multiplicity 2 tangent planes and rectifiability of multiplicity 2 branch sets of stable minimal hypersurfces (latter joint work with B. Krummel) and
2. regularity, up to a closed set of parabolic Hausdorff dimension 1, of 1-dimensional Brakke flows admitting no static tangent flows of density ≥ 2 (joint work with Y. Tonegawa).

2014/07/07, 11:45 — 12:45
Wilking, Münster University

Some new curvature identities and applications to Ricci flow invariant curvature conditions

Various problems in Ricci flow in higher dimensions are related to the lack of the understanding of the Ricci flow ODE, which is an ordinary differential equation in the space of algebraic curvature tensors: $R' = R^2 + R^{\#}$. We express the right hand side as an integral over a much simpler quadratic curvature expression in the case that R is Ricci flat. This is one of the ingredients which allows to show that in large dimensions the curvature condition $(n-2+x)\|R\|^2 \leq \|\operatorname{Ric}(R)\|^2$ combined with $\operatorname{scal} \geq 0$ is Ricci flow invariant for $x \in [0, n]$.

2014/07/07, 15:00 — 16:00
Zhou, Massachusetts Institute of Technology

Min-max minimal hypersurface in manifold of positive Ricci curvature

We will discuss the shape of the min-max minimal hypersurface produced by Almgren-Pitts corresponding to the fundamental class of a Riemannian manifold (M^{n+1}, g) of positive Ricci curvature with $2 \leq n \leq 6$. We characterize the Morse index, volume and multiplicity of this min-max hypersurface.